*Rejoinder on "Conjectures on exact solution of three-dimensional (3D) simple orthorhombic Ising lattices"**

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Abstract

It is shown that the arguments in the reply of Z.-D. Zhang defending his conjectures are invalid. His conjectures have been thoroughly disproved.

After all the discussion about his paper [1, 2, 3, 4, 5], Zhang seems to have only one real issue left with the two comments [2, 5], still wrongly believing [6] that the free energy of the three-dimensional Ising model is not analytic at $\beta \equiv 1/(k_{\mathrm{B}}T) = 0$, $H = 0$. His further arguments are irrelevant or dealt with adequately in [2, 5].

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His objection that Gallavotti and Miracle-Solé set $\beta = 1$ is not valid. One often uses dimensionless parameters $K_i = \beta J_i$, $(i = 1, 2, 3)$, $h = \beta H$. Equivalently, one can absorb the β into the coupling constants, setting $\beta = 1$, so that $K_i \equiv J_i$, $h \equiv H$. Infinite temperature is then the limit $K_i \to 0$, $h \to 0$, such that all ratios are kept fixed.[1]

His point on [7] is also not valid. Writing $z \equiv \exp(-2\beta H)$, see eq. (23) of [7], and keeping βH fixed in the limit $\beta \to 0$, the Ising model partition function on an arbitrary lattice with N sites becomes $Z = (z^{1/2} + z^{-1/2})^N$, so that all infinite-temperature zeros of Z occur at $z = -1$, i.e. for the purely imaginary magnetic field [7] $H = \pm i\pi k_{\mathrm{B}}T/2 = \pm i\infty$. There is no $T = \infty$ singularity at $H = 0$, $z = 1$.[2]

References

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[7] T.D. Lee and C.N. Yang, Phys. Rev. **88** (1952) pp. 410–419.

[8] C.N. Yang and T.D. Lee, Phys. Rev. **88** (1952) pp. 404–409.

[1] The reduced free energy per site βf is often rewritten $\beta f = \phi(\{K_i\}, h) = \phi(\{\beta J_i\}, \beta H)$ with some function ϕ. Setting $\beta = 1$ is no loss of generality, as one can easily restore the β-dependence by the replacements $J_i \to \beta J_i$, $H \to \beta H$, $f \to \beta f$.

[2] In Zhang's paper [1], $H \equiv 0$, $z \equiv 1$, $Z = 2^N$ for any lattice with N sites at $T = \infty$, which is far from $z = -1$. Hence, the infinite-system dimensionless free energy βf is analytic at $\beta = 0$ also by the general theory of Yang and Lee [8].