

## A Limit

Find the limit  $\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)^{\frac{1}{x^2}}$ .

$$\begin{aligned} \lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)^{\frac{1}{x^2}} &= \lim_{x \rightarrow 0} \left( 1 + \frac{\tan x - x}{x} \right)^{\frac{x}{\tan x - x} \frac{\tan x - x}{x^3}} \\ &= e^{\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}} = e^{\lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{3x^2}} = e^{\lim_{x \rightarrow 0} \frac{2\sec^2 x \tan x}{6x}} = e^{\frac{1}{3}}. \end{aligned}$$