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Controllable preparation of entangled coherent states with superconducting system

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ABSTRACT

Utilizing a current-biased Josephson junction (CBJJ) as a tunable coupler for superconducting transmission line resonators (TLRs), we propose a potentially practical scheme to create entangled coherent states of the two TLR modes. Then, the influence of TLRs decay on the prepared entangled states is analyzed. And an interesting phenomenon that even entangled coherent states are robustness against decay with small α is found. At last, the experimental feasibility and the challenge of our schemes have been discussed.

1. Introduction

The charming entangled states are playing an important role in quantum world [1]. They are not only helpful in quantum mechanics to prevail over local hidden theory [2], but also valuable in quantum information process (quantum teleportation [3], quantum dense coding [4], and quantum cryptography [5]). Hence, seeking a graceful way to generate entangled states is still a hot topic. Up to now, based on cavity quantum electrodynamics (QED), dazzle color multicolored theoretical schemes have been proposed to prepare entangled states, which include two-qubit entangled states [6,7], Greenberger–Horne–Zeilinger (GHZ) states [8], W-type states [9], cluster states [10], and entangled coherent states [11–13]. Among these states, entangled coherent states have attracted much attentions because of its robustness against single-particle decoherence and important applications in quantum information processing [14]. However, how to generate entangled coherent states become an interesting gambit.

On the other hand, the solid circuit superconducting devices (Cooper-pair boxes, Josephson junctions, and SQUID) were proposed as candidates to serve as the qubits for a superconducting quantum computer [15], due to its advantage in design flexibility, large-scale integration, and compatibility to conventional electronics. Therefore, this is a fascinating study field in the quantum information world. The coherent control of macroscopic quantum states in a single-Cooper-pair box has been realized [16]. The detection of geometric phases in superconducting qubit has been reported [17]. And the circuit QED system was wide utilized in generation of all kinds of discrete entangled states [18]. Soon afterwards, Zhang et al. [19] proposed a practically feasible scheme to generate entangled coherent states by coupling two superconducting *LC* modes with a flux qubit. However, this method has some disadvantages: (i) The coupling between *LC* circuit and flux qubit is reduced by mutual inductance, which is sensitive to noise disturbance. (ii) Three-step operations were required to produce entangled coherent states. However, the onerous operations are huge challenge with current experimental technology. (iii) Through measuring the flux qubit states, the maximum entanglement states of the two *LC* coherent modes were obtained, therefore, the outcome probability is only 50%.

In this Letter, we proposed an alternative scheme to generate entangled coherent states using a CBJJ to induce the interaction of two TLRs. This scheme avoids the disadvantages of Ref. [19]. In addition, one of the favorable feature of our scheme is that the structure is very simply and has the feasibility with current experimental condition. Also, the effect of the TLRs decay on the generation of even entangled coherent states and odd entangled coherent states were discussed, respectively.

2. System setup

The controllable coupling of two identical superconducting TLRs via a CBJJ acts as a tunable coupler has been proposed [20]. However, in the realistic experiment, it is difficult to prepare twinborn TLRs. So, we do not require two identical TLRs in our scheme (see Fig. 1(a)).

The CBJJ. The quantum properties of the CBJJ have been well studied in Ref. [21]. The CBJJ can be modelled by a fictitious particle moving in a tilted washboard potential well. When the junction

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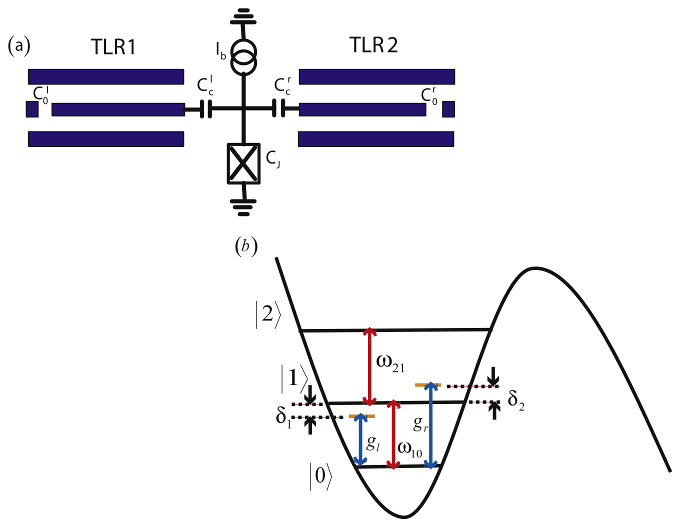

Fig. 1. (Color online.) (a) The tunable coupled system of two TLRs are connected to a CBJJ from left to right by coupling capacitors C_c^i. (b) The energy level configuration of the CBJJ. Two TLR modes interact with level transitions $|0\rangle \leftrightarrow |1\rangle$, respectively.

bias current I_b is close to the critical current I_c, the anharmonic potential can be well approximated by a cubic potential, one can construct a three-level quantum system (see Fig. 1(b)). In this Letter, we only choose the two lowest energy states. The reduced Hamiltonian of a single CBJJ is given by (in units of $\hbar = 1$)

$$H_q = \frac{1}{2}\omega_{10}\sigma_z, \tag{1}$$

where $\omega_{10} \simeq 0.9\omega_p$ is the frequency difference of the two lowest energy levels, $\omega_p = \sqrt[4]{(2 - 2I_b/I_c)(2\pi I_c/\Phi_0 C_J)}$ expresses the plasma oscillation frequency at the bottom of the well [22], $\Phi_0 = h/2e$ is the flux quantum, C_J is the junction capacitance; and $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ is the Pauli operator.

The TLR. A single TLR can be well described by a series of inductors with each node capacitively connected to the ground. And the TLR should be coupled to the external subsystem by capacitors C_0^i. The Hamiltonian of the two individual TLRs can be written as

$$H_c = \omega_l a_l^\dagger a_l + \omega_r a_r^\dagger a_r, \tag{2}$$

where $a_l^\dagger$ and $a_r^\dagger$ are the creation operators of full wave modes in the left and right TLRs, respectively. $\omega_i \approx \omega_0^i(1 - \epsilon_1^i - \epsilon_2^i)$ $(i = l, r)$ expresses the frequency of the TLRs, where $\omega_0^i = 2\pi/(L_i\sqrt{F_i C_i})$, L_i is the length of TLRs, F_i and C_i are the inductance and capacitance per unit length, respectively. And denoting $\epsilon_1^i = C_0^i/L_i C_i$, $\epsilon_2^i = C_c^i/L_i C_i$.

The combined system. The two-mode JC Hamiltonian can describe the interaction of TLR–CBJJ–TLR. Therefore, under rotating-wave approximation, the interaction Hamiltonian between the CBJJ and the TLRs can be written as

$$H_{int} = g_l\left(a_l\sigma_+ + a_l^\dagger\sigma_-\right) + g_r\left(a_r\sigma_+ + a_r^\dagger\sigma_-\right), \tag{3}$$

where the coupling factor $g_i = \omega_i C_c^i/[2L_i C_i(C_J + 2C_c^i)]^2$ and $\sigma_+ = \sigma_-^\dagger = |1\rangle\langle 0|$.

3. Generation of entangled coherent states

In the interaction picture, the system Hamiltonian reads

$$\begin{aligned} H_I = {} & g_l\left(a_l^\dagger\sigma_- e^{-i\delta_1 t} + a_l\sigma_+ e^{i\delta_1 t}\right) \\ & + g_r\left(a_r^\dagger\sigma_- e^{-i\delta_2 t} + a_r\sigma_+ e^{i\delta_2 t}\right), \end{aligned} \tag{4}$$

where the detuning $\delta_1 = \omega_{10} - \omega_l$, $\delta_2 = \omega_{10} - \omega_r$. If the detuning $\delta_{1,2}$ is sufficiently large, i.e. $|\delta_{1,2}| \gg g_i$, using adiabatically eliminated method, the effective Hamiltonian of Eq. (4) can be written as

$$\begin{aligned} H_e = {} & \lambda_1\left(a_l a_l^\dagger\sigma_+\sigma_- - a_l^\dagger a_l\sigma_-\sigma_+\right) \\ & + \lambda_2\left(a_r a_r^\dagger\sigma_+\sigma_- - a_r^\dagger a_r\sigma_-\sigma_+\right) \\ & + \lambda_3\left[a_r a_l^\dagger\sigma_+\sigma_- e^{-i(\delta_1-\delta_2)t} - a_r^\dagger a_l\sigma_-\sigma_+ e^{i(\delta_1-\delta_2)t}\right] \\ & + \lambda_4\left[a_l a_r^\dagger\sigma_+\sigma_- e^{i(\delta_1-\delta_2)t} - a_l^\dagger a_r\sigma_-\sigma_+ e^{-i(\delta_1-\delta_2)t}\right], \end{aligned} \tag{5}$$

where the parameters $\lambda_1 = g_l^2/\delta_1$, $\lambda_2 = g_r^2/\delta_2$, $\lambda_3 = g_l g_r/\delta_1$, and $\lambda_4 = g_l g_r/\delta_2$. We assume initial state of the CBJJ is prepared in the state $|0\rangle$, the effective Hamiltonian of describing evolution of the TLRs is

$$\begin{aligned} H_e = {} & \lambda_1 a_l^\dagger a_l + \lambda_2 a_r^\dagger a_r \\ & + \lambda_3 a_r^\dagger a_l e^{i(\delta_1-\delta_2)t} + \lambda_4 a_l^\dagger a_r e^{-i(\delta_1-\delta_2)t}. \end{aligned} \tag{6}$$

Eq. (6) can be divided two parts: $H_e^0 = \lambda_1 a_l^\dagger a_l + \lambda_2 a_r^\dagger a_r$ and $H_e^I = \lambda_3 a_r^\dagger a_l e^{i(\delta_1-\delta_2)t} + \lambda_4 a_l^\dagger a_r e^{-i(\delta_1-\delta_2)t}$. We implement a unitary transformation $e^{-iH_e^0 t}$ on H_e^I. When the conditions $|\lambda_3| = |\lambda_4| = \lambda$ and $\lambda_2 - \lambda_1 + \delta_1 - \delta_2 = 0$ are satisfied, the H_e^I becomes

$$H_e^I = \lambda\left(a_l^\dagger a_r + a_l a_r^\dagger\right). \tag{7}$$

Eq. (7) is a generator of SU(2) coherent state. In order to generate entangled coherent states, we assume the initially left TLR mode a_l is prepared in the even coherent states (or odd coherent states) $|\Psi_+(0)\rangle = M_+(|\alpha\rangle + |-\alpha\rangle)$ $(|\Psi_-(0)\rangle = M_-(|\alpha\rangle - |-\alpha\rangle))$, with the normalization factors $M_\pm = (2 \pm 2e^{-2|\alpha|^2})^{-1/2}$, and the right TLR mode a_r is in the vacuum state $|0\rangle$, i.e. $|\Psi_\pm(0)\rangle = M_\pm(|\alpha\rangle \pm |\alpha\rangle)|0\rangle$. After an interaction time t, we obtain the entangled coherent states

$$\left|\Psi_\pm(t)\right\rangle = M_\pm\left(|\alpha\cos\lambda t, -i\alpha\sin\lambda t\rangle \pm |-\alpha\cos\lambda t, i\alpha\sin\lambda t\rangle\right). \tag{8}$$

Using the concept of concurrence for bipartite entangled nonorthogonal states [23], the concurrence of Eq. (8) is given by [24]

$$C_\pm = \frac{\sqrt{(1 - e^{-4|\alpha|^2\cos^2\lambda t})(1 - e^{-4|\alpha|^2\sin^2\lambda t})}}{1 \pm e^{-2|\alpha|^2}}. \tag{9}$$

When the evolution time $t = (2n+1)\pi/4\lambda$ $(n = 0, 1, \ldots)$, the C_- is one, the maximally odd entangled coherent states can be obtained with the random α, however, for the even entangled coherent states, the $C_+ = (1 - e^{-2|\alpha|^2})/(1 + e^{-2|\alpha|^2})$ increases with the increasing of $|\alpha|$.

4. The effect of cavity decay on the fidelity of entangled coherent states

We have shown how to entangle two TLR modes through a CBJJ in Section 3. As is known to all, for a realistic quantum system, the system decay plays an important role. In this section, we discuss the effect of the TLR decay on the generation of entangled coherent states. The evolution of the decay system is described by the master equation

$$\begin{aligned} \dot{\rho} = {} & -i\left[H_e', \rho\right] + \frac{\kappa_a}{2}\left(2a_l\rho a_l^\dagger - a_l^\dagger a_l\rho - \rho a_l^\dagger a_l\right) \\ & + \frac{\kappa_b}{2}\left(2a_r\rho a_r^\dagger - a_r^\dagger a_r\rho - \rho a_r^\dagger a_r\right), \end{aligned} \tag{10}$$

where κ_a and κ_b are the decay rate of the left and right TLR, respectively.

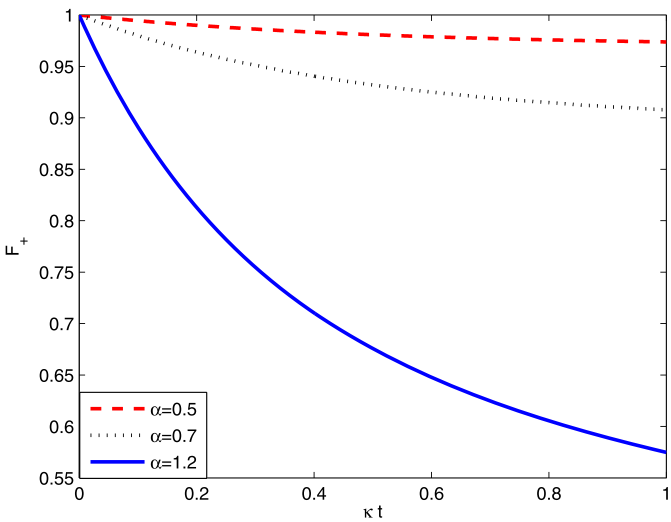

Fig. 2. (Color online.) The fidelity of the even entangled coherent state $|\Psi_+(t)\rangle$ versus κt, with different α.

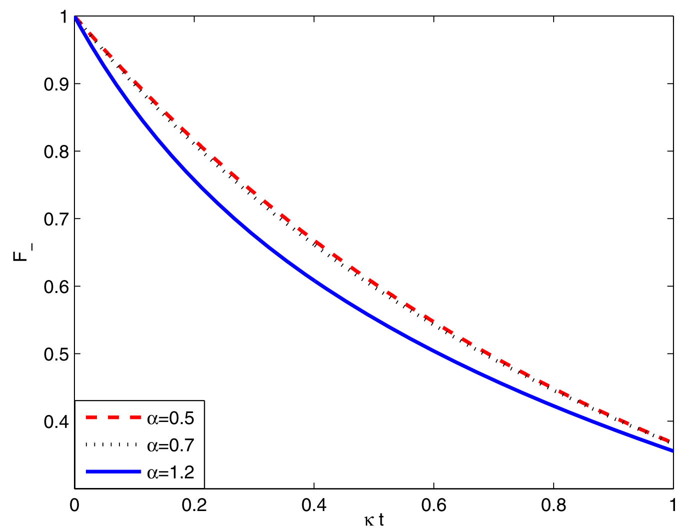

Fig. 3. (Color online.) The fidelity of the odd entangled coherent state $|\Psi_-(t)\rangle$ versus κt, with different α.

Fidelity is a direct measure to characterize how accurate generate entangled coherent states. In order to simple, we assume the two TLR modes have the same decay rate, i.e. $\kappa_a = \kappa_b = \kappa$. The analytic solution of the fidelity can be obtained. For the even coherent states $|\Psi_+(t)\rangle$, the fidelity is given by

$$\begin{aligned}F_+ &= \langle\Psi_+(t)\big|\rho\big|\Psi_+(t)\rangle\\ &= M_+^4\big[2\big(1+e^{-2|\alpha|^2(1-e^{-\kappa t})}\big)\big(e^{-|\alpha|^2(1-e^{-\frac{\kappa t}{2}})^2}\\ &\quad+e^{-|\alpha|^2(1+e^{-\frac{\kappa t}{2}})^2}\big)+4e^{-|\alpha|^2(3-e^{-\kappa t})}\\ &\quad+4e^{-|\alpha|^2(1+e^{-\kappa t})}\big].\end{aligned} \tag{11}$$

The relation between the fidelity F_+ of even entangled coherent states and κt, with different α was shown in Fig. 2. We find that fidelity decreases fast while increasing α. For the odd coherent states $|\Psi_-(t)\rangle$, the fidelity reads

$$\begin{aligned}F_- &= \langle\Psi_-(t)|\rho|\Psi_-(t)\rangle\\ &= M_-^4\big[2\big(1+e^{-2|\alpha|^2(1-e^{-\kappa t})}\big)\big(e^{-|\alpha|^2(1-e^{-\frac{\kappa t}{2}})^2}\\ &\quad+e^{-|\alpha|^2(1+e^{-\frac{\kappa t}{2}})^2}\big)-4e^{-|\alpha|^2(3-e^{-\kappa t})}\\ &\quad-4e^{-|\alpha|^2(1+e^{-\kappa t})}\big].\end{aligned} \tag{12}$$

And we have plotted the fidelity F_- of odd entangled coherent states as a function of κt with different α (Fig. 3). Comparing with even entangled coherent states, one can see from Fig. 3 that the fidelity of odd entangled coherent states is insensitive to the α. So, generation of the small α corresponding to even entangled coherent state has the relatively high fidelity. Also, the fidelity decreases accordingly with the increasing of decay rate κ was shown above figures, therefore, the high-Q TLR is preferred in our scheme.

5. Discussion and conclusion

We briefly address the experimental feasibility of the proposed scheme with the parameters already available in current experimental setups. The coupling strength g_i of the TLR–CBJJ can be effectively controlled by adjusting the bias current I_b. Hence, the effective coupling constant λ of the two TLRs is modulated with the changing g_i. Ref. [25] has reported the parameters of the TLR: eigenfrequency $\omega_r/2\pi = 10$ GHz, quality factor $Q = 1\times10^5$, and decay rate $\kappa = 0.1$ MHz. The parameters of the CBJJ are $C_J = 5.8$ pF, $I_c = 140$ μA, $I_b \approx 0.99 I_c$, and coupling capacitance $C_c^i = 6$ fF [20]. With the above parameters the transition frequency of two lowest states is $\omega_{10}/2\pi = 2$ GHz and the coupling constant is $g_i/2\pi = 17$ MHz. The large detuning conditions were satisfied. We take the integer $n = 0$, the time t_p of production of perfect entangled coherent states $t_p = 2.2\times10^{-5}$ s. Thus we predict that our proposal might be experimentally realized with current technology.

In summary, we have proposed a scheme to generate entangled coherent states by a CBJJ coupling with two TLRs. And, the effect of the TLR losses on the generation of entangled coherent states was discussed. Also, we found that even entangled coherent states have high fidelity with small α. In addition, we have analyzed the experimental feasibility of our scheme with current technology.

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