Book Reviews

With this issue, the IEEE Automatic Control Group will publish some reviews of books in the control field and related areas. Readers are invited to send comments on these reviews for possible publication in the Correspondence section of these TRANSACTIONS. The G-AC does not endorse the opinions of the reviewers.

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Differential Games—R. Isaacs. (New York: Wiley, 1965, 384 pp.)

I. A Few Amazing Facts

This is a remarkable, in fact astounding, book. It is remarkable in several ways:

1) About half the book (and this includes 90 percent of the theory) was completed before 1956. It thus anticipated practically all the important control-theoretic results known to date by nearly a decade. In fact, in several instances, the book went beyond present control theory knowledge.

2) The book was written by an author who apparently has never heard of dynamic programming or the principle of optimality (although he uses the idea freely); who has only learned the name "Pontriagin's maximum principle" after the book was completed in 1963; and whose conception of what constitutes the classical calculus of variation is rather elementary judging from the remarks he made on the subject in the course of the book.

3) Between the time most of the book was written and its publication, a span of about nine years, the author, apparently through his own choice, made no effort to publish his ideas; nor did he make any effort to learn the parallel development in control theory during this time. Consequently, all the notations and terminology are uniquely his own.

4) In these days of increasing efforts towards abstraction and generalization, this is a book devoted to solving specific problems and to showing that besides generality it is also *interesting*, *hard*, and *rewarding* work to solve specific problems. Over half the book consists of specific examples of differential games solved with varying detail by using mostly the theory developed plus ingenuity, geometrical intuition, and common sense applications of mathematical analysis. These are shining examples of the abundant insight and mastery which the author has on the subject matter.

Before this reviewer substantiates (or rather, *defends*) these claims, it is necessary at this time to answer one question for the control engineering audience.

II. What Is a Differential Game?

Consider the dynamic system with two sets of control variables u and v.

$$\dot{x} = f(x, u, v, t); \qquad x(t_0) = x_0 \tag{1}$$

and the criterion of minimizing

$$J = \phi(x(T), T) + \int_{t_0}^{T} L(x, u, v, t)dt \tag{2}$$

and the constraints

$$\psi(x(T), T) = 0 \qquad (q \text{ equations, } T \text{ free}) \tag{3}$$

$$u(t) \in U \qquad v(t) \in V. \tag{4}$$

Now define for convenience

$$\Phi(x(T), T) = \phi(x(T), T) + \nu^T\psi(x(T), T) \tag{5}$$

$$\mathcal{H}(x, u, v, \lambda, t) = \lambda^T f + L. \tag{6}$$

Then, it is well known (at least reasonably) today that the necessary conditions for $u(t)$ and $v(t)$ to be optimal are:

i) Euler-Lagrange equations

$$\begin{cases} \dot{x} = f(x, u, v, t) = \mathcal{H}_\lambda \\ \dot{\lambda} = -\mathcal{H}_x, \end{cases} \tag{7}$$

ii) Transversality conditions at T

$$\lambda^T(T) = \Phi_x\big|_T; \qquad \psi(x(T), T) = 0 \tag{8}$$

$$\mathcal{H}(T) = -\Phi_T \tag{9}$$

iii) Minimization condition

$$\mathcal{H}^0(x, \lambda, t) = \min_{u\in U} \min_{v\in V} \mathcal{H}(x, u, v, \lambda, t) \tag{10}$$

which implies

$$u = k_1(x, \lambda, t)$$
$$v = k_2(x, \lambda, t). \tag{11}$$

Furthermore, we have the Hamilton-Jacobi-Bellman partial differential equation for the optimal J.

$$J_t^0(x, t) = \min_{u\in U} \min_{v\in V} \mathcal{H}(x, u, v, J_x^0, t) \tag{12}$$

$$J^0(x(T), T) = \phi(x(T), T) \text{ on the terminal surface defined by (3).} \tag{13}$$

If a unique $J^0(x, t)$ solution of sufficient smoothness exists for (12), then we can identify $\lambda = J_x^0$ (on the optimal trajectory), and $u = k_1(x, J_x^0, t)$, $v = k_2(x, J_x^0, t)$ are the *optimal* control laws.

Now suppose instead of cooperating in the minimization of J, the set of control variables v are actually under the direct manipulation of an adversary whose interest is to maximize J, i.e., we wish to determine a minimax. Then, it is reasonable to conjecture that the necessary conditions (i–iii) remain unchanged except for replacing $\min_{v\in V}$ in (10) and (12) by $\max_{v\in V}$. The theory of differential games says that this conjecture is *essentially* true. Lest the reviewer misled the reader to think that the subject of differential games is thus trivial from control theoretic viewpoint, let him hasten to point out the italicized word in the foregoing sentence. A rough idea of the *theoretical* complication introduced by the presence of an opposing control variable can be obtained by pointing out that, in the comprehensive mathematical paper by Berkovitz [6] on this subject, 17 *pages* are required for mathematical assumptions and definitions before a proof of the conjecture can be attempted. Even so, the rigorous proof so far only covers for the case $q=1$ in (3). On the other hand, while the author attempts only for "engineering rigor" in the book, he has successfully demonstrated by way of solving specific problems the practical complications that are inherent in differential games. Generally speaking, problems such as

a) existence of solutions (e.g., can capture occur?)
b) uniqueness of solutions (multiple solutions yielding same value)
c) singular solutions, when (10) is identically satisfied and does not imply (11)
d) miscellaneous, conjugate points, switching surfaces (discontinuous J_x^0), etc.

occur in differential game problems *as a rule* rather than as exceptions. Thus, solution of a DG problem requires more than a routine application of conditions (7)–(13).

No constraint on x i.e.: x ∈ B See e.g. Chap. 6 of L.S. Pontryagin et. al. "The Math. Theory of Optimal Processes", Interscience, 1962.

III. Chapter Titles and Brief Descriptions

Let us take a look at the chapter headings:

For the control theory audience, chapters 1 and 2 represent familiar material concerning topics such as the concept of state space, the equivalence between the Mayer and Bolza formulations, etc. Chapter 3 proceeds to solve discrete time and discrete state games via the principle of optimality of the backwards sweep method of dynamic programming, although nowhere does the author use this terminology. Chapter 4 represents the development of the theory [Eqs. (7)–(13), this review]. Once again we must emphasize that these results were derived in 1955! Two derivations are given, both of which represent different applications of the optimality principle. Sufficiency conditions [statement following (13), this review] are proved under the name of "Verification Theorem." The next eight chapters consist mainly of examples illustrating the application of the theory in Chapter 4 and various interesting complications [items a)–d) in Section II] that can arise. The reviewer knows of no source within control theory literature where so many intriguing examples have been so carefully worked out and illustrated with diagrams and pictures. The pedagogical values of these problems are immense. The book is a very nice example of learning and discovering generality by doing specific problems.

IV. Battle Plan for Reading the Book

Despite all the impressive and good things that may be said for the book, it is basically unreadable for anyone who is not already familiar with most of the control-theoretic facts of life stated in Section II. Even experienced control theorists have to make a conscientious effort to divorce themselves from established notations and to adjust to the author's writing style which makes skipping from chapter to chapter very difficult. To save future readers from additional agony, the reviewer has provided the following dictionary and battle plan for reading the book.

Isaacs-to-Control Dictionary

Isaacs	*Control Theory*
a) State variables x	x
b) Control variables ϕ and ψ	u and v
c) Value of the game $V(x)$	Optimal return function $V(x)$
d) Main Eq. (1)	H-J-B P.D.E. (12), this review
e) Main Eq. (2)	$V_t + H^0 = 0$
f) Retrospective path equations	Euler-Lagrange (7)
g) s_i or V_i	$\lambda_i(t)$ multiplier function
h) $H(s_i)$	$\phi(x(T), T)$
i) C the terminal surface	$\psi(x(T), T) = 0$
j) Initial Conditions for RPE stated on page 84, first nine lines of the text	Transversality conditions (8) and (9)
k) Universal surfaces	Singular surfaces
l) Transition surfaces	Switching surfaces
m) Barrier	Boundary of domain of controllability
n) $\sum V_i f_i + G$	$\mathcal{H} = \lambda^T f + L$
o) Semipermeable surface	Surface of constant $V(x)$ (for $L=0$ case)
p) $A = \sum \alpha_i V_i = 0$	$\mathcal{H}_u = 0$
$B = \sum \beta_i V_i = 0$	$\mathcal{H} = \text{constant}$
$C = \sum \gamma_i V_i = 0$	$(d/dt)\mathcal{H}_u = 0$
$D = \sum (\partial\gamma_i/\partial x_j)\alpha_j - (\partial\alpha_i/\partial x_j)\gamma_j) V_i \leq 0$	$(\partial/\partial u)(d^2/dt^2)\mathcal{H}_u \leq 0$

This gives some idea of the notational red tape that one has to cope with. The reviewer suggests a battle plan to read the book for minimax efficiency (i.e., the differential game of extracting useful information from the book).

A. The Battle Plan

1) Start with Chapter 4 and attempt identification of the dictionary items a) through j). The last item should offer a real challenge to the reader.

2) Attempt to understand the solution of the isotropic rocket example in Chapter 5.

3) Proceed to Chapter 9 for the extension of the isotropic rocket example using the translation m and the concept of the conjugate point condition in [1].

4) Return to Chapter 7 and try to decode the mysterious A, B, C, and D in item p). However, to accomplish this, one should be aware of these recently established facts on singular problems:

a) In three-dimensional state space, the conditions $H^0 + V_t = 0$; $H_u = 0$; and $(d/dt)H_u = 0$ determine the singular surface. Otherwise the singular surface exists in $x-\lambda$ space in general.
b) By reducing the number of state variables (i.e., restricting the problem on the singular surface), a singular problem can always be made nonsingular ([2], Isaacs, Sec 7.12).
c) A necessary condition for optimality of a singular arc is $(\partial/\partial u)(d^2/dt^2)\mathcal{H}_u \leq 0$ [3].
d) The foregoing items generalize to $(-1)^n(\partial/\partial u)(d^{2n}/dt^{2n})\mathcal{H}_u \geq 0$ [4].

While doing these steps, one may reflect on the fact that Isaacs wrote Chapter 7 some time between 1955 and 1962, which is long before the datings of [2]–[4].[1]

5) If the reader survives this far, then he can tackle Chapters 6 and 10, and Section 9.4 on the envelope barriers. These materials are unique with differential games and have no counterpart in control theory.

6) Lastly, read the various unsolved but enticing speculations in Chapter 12.

7) Do any of the examples in the book using control-theoretic notations. Any reader who follows this plan will be more than amply rewarded.

It is also worthwhile to point out one operational difference between differential game problems and control theory problems. The number of state variables involved in the criterion function is usually a small fraction of the total number of state variables in a DG problem. For example, the problem of minimizing the time to drive the first component of the state vector to zero for a linear dynamic system is considered trivial and unrealistic in control theory. On the other hand, the problem of minimaximizing the time for intercept of one dynamic system by another in one component is a legitimate

[1] To be fair to the contribution of control theorists it should be pointed out that there is a subtle but conceptually important difference between the universal surface of Isaacs and the singular surfaces of calculus of variations. Briefly, universal surfaces are only a subset of all possible singular surfaces. The equivalence between D and $(\partial/\partial u)(d^2/dt^2)\mathcal{H}_u \leq 0$ is only operational. The conceptual basis of derivation is different. The generalization of "D" condition (Isaacs, p. 188) leads to results slightly different from condition "d" and is applied differently. However, these are highly technical points and will not be discussed here. Interested readers are referred to [10]. The reviewer is indebted to Dr. H. J. Kelley for this footnote.

DG problem of value. Yet, it is just as easy to solve the DG problem as its control theory counterpart [5]. Thus, DG problems are actually easier in this sense. This also explains the fact that so many interesting examples where explicit feedback strategies can be worked out are present in the book.

V. Specific Remarks

1) The derivation of the Euler-Lagrange equations from the HJB P.D.E. is so nonrigorous as to be incomplete (p. 80, last line).

2) Too many footnotes are used in the book. A typical sampling of 63 pages discloses 45 footnotes.

3) Some of the research problems (i.e., unsolved problems) in the book have well-known solutions in control theory.

4) It also helps to be an English major when reading the book. On the first 44 pages one finds words such as, "adumbration, assuage, aliquot, outre, congeries, jejune, hoary, desultory, consort, pristine, and proscription."

5) One inevitable question naturally arises—If Dr. Isaacs knew all that is in the book nine years ago, then what does he know now?

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Nonlinear Magnetic Control Devices—William A. Geyger. (New York: McGraw-Hill.)

The author has set down a number of reasons for the writing of his book. These are:

1) A compact survey on nonlinear magnetic devices in one book.
2) A reference book to permit "self-instruction" in the theory of generation of nonlinear magnetic devices.
3) "That the best way to learn about such devices is to use them," and as such, stress the practical experimental approach. This brings one face-to-face with the instrumentation problems.
4) Derivation of the operation of nonlinear magnetic devices by use of waveforms and word description with a minimum of rigorous formal operational mathematics.
5) The contents intended for practical use.

Having set down the foregoing objectives, the author then proceeds to cover in the Introduction and the fourteen chapters the broad field of magnetic devices from the study of magnetic materials to the development of ac and dc saturable reactor power and signal amplifiers, error detectors, frequency multipliers, counters, modulators, and culminating in the flux gate magnetometer. In this he was successful.

The author develops the buildups of his story through the sequence of fourteen chapters. They are

1. Static and dynamic hysteresis loops of magnetically soft core materials.
2. Combination of ac saturated nonlinear inductors and linear impedance elements.
3. The application of saturating-core circuits to representative practical problems.
4. Principles and characteristics of ordinary saturable reactors.
5. Application of ordinary saturable reactors in dc instrumentation.
6. Two-core saturable devices utilizing feedback techniques.
7. Four-core saturable-reactor devices utilizing feedback techniques.
8. Magnetic frequency multipliers with single-phase supply.
9. Magnetic frequency multipliers with three-phase power supply.
10. Magnetic frequency multipliers of the shock-excitation type.
11. Switching-transistor magnetic-coupled multivibrators and their applications.
12. Magnetic modulators.
13. Principles and characteristics of flux-gate magnetometers.
14. Typical applications of flux-gate magnetometers.

A large number of specific classes of magnetic devices have been covered. In most cases the author begins with a description of the operation and then proceeds to the practical circuit. The characteristics of the practical devices are described giving their electrical properties, and also the environmental effects upon the critical magnetic components are considered. The limitations are brought to light.

The development of special instrumentation to measure the characteristics is developed in some cases and recognized in others, for an example, at the outset the need of a "Ferrotracer" for plotting B-H curves on graph paper was recognized as a necessary instrument to permit ease in development dealing with core materials.

Next the author lists known work in progress by others in the given field.

Each chapter is concluded with two- to four-page references, listing national and international contributions.

The author has done a formidable task in coming to grips with the magnetic modulators. The gain and zero drifts and approaches to minimize these are defined and also the considerations of input and output impedance boundaries are recognized. The fundamental and second harmonic types are considered. Effects of stray fields are recognized and resolved. In some cases basic new design approaches are implemented.

The chapters (13 and 14) on the flux-gate magnetometers represent a contribution to the literature. The expanding interest in the exploitation for commercial survey and on the impact of economic growth forms a strong incentive for greater expansion. The author stresses the indirect contribution in oil survey. In the historical build-up the significance of the airborne application for military underwater detection was illuminated. To the reviewer it was odd that no mention was made of the flux-gate compass. The application of the flux-gate compass with the Magnesyn repeaters along with the associated servo amplifiers using magnetic saturable transformers reached in WW II a level estimated in the many, many thousands (B-29's, B-17's, etc.). The use of the flux-gate compass permitted the use of azimuth stabilization to be incorporated in search radar systems.

Critical Comments

In those cases where the author mentions the transient response or the time constant, it is usually referred to in cycles of the carrier. It is believed that to the controls engineer, pictures of the transient response of the magnetic amplifier with the driving impedance and load combined should have been included. Also, the envelope of the carrier under various load conditions such as reactive and resistive over reasonable dynamic range conditions is of fundamental importance to the control engineer.

In the section covering magnetic amplifier integrators and differentiators a few words on the quality or "goodness" would have been appropriate.